Diode Transient Response: A Re-Evaluation

James K.E. Tunaley[1,a)]
[1]London Research and Development Corporation,
114 Margaret Anne Drive, Ottawa, ON, Canada, K0A 1L0.

The transient response of a p – n junction has been analyzed by Lax and Neustadter. They provide the theory for reverse recovery as well as plots of the junction voltage and current as a function of time and of the external circuit resistance. The theory is based on a diffusion model. However, there are errors in some of the equations that add sufficient uncertainty to render a reproduction of the analysis non-trivial. The object of this work is to verify their treatment and to obtain plots by straightforward numerical means.

Lax and Neustadter[1] have analyzed the transient response of a diode when an applied voltage is rapidly switched from the forward to the reverse direction (see Figure 1). They describe the basic theory, which is an adaptation of the theory for heat conduction described by Carslaw and Jaeger[2]. The transient response relies on an expression for the diode current as a function of time and junction voltage. This is then combined with a simple Ohm's law equation describing the current as a function of the circuit resistance, the junction voltage and the reverse applied voltage. The first of these expressions is non-linear by virtue of Boltzmann relation (reflected in the diode characteristic). Unfortunately, their paper contains a few typing errors (e.g. their equation (3)) and it has been necessary to repeat the calculations in detail and correct these before attempting to reproduce and extend their results.

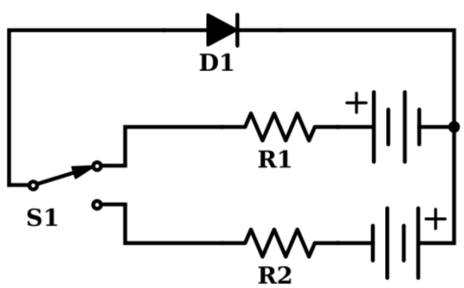

Figure 1. Reverse recovery circuit: Switch in forward position.

The diode model is a planar p – n junction. The concentration of holes is much higher than that of electrons and the electron contribution is ignored. The junction is very abrupt so that the calculations are confined to the n-type semiconductor material, which is considered to be one-dimensional and semi-infinite. The hole density at the edge of the n-type material is given by the product of the equilibrium density, p_n, and a Boltzmann factor; the hole density within the n-type material is controlled by diffusion and recombination. Lax and Neustadter, in their equation (6), provide the hole density, $p(x,t)$, as a function of position, x, and time, t. This equation appears to be correct. It arises from well-known solutions of the diffusion equation described by Carslaw and Jaeger[2]. The current through the junction is proportional to the rate of change of hole density with x at $x = 0$.

The hole density in the n-type semiconductor consists of three terms; the first two contain an integral. The first term involves the junction voltage as a function of time and the second term involves the junction voltage at $t = 0$. The third term is just the constant p_n.

To simplify matters from the start, we use normalized variables, X and T, which are related to the actual variables by factors L and τ, respectively. These parameters are the characteristic diffusion length and the recombination time. The first term is given by:

$$p(X,T) = p_n \frac{Xe^{-T}}{2\pi^{1/2}} \int_0^T G(\lambda) \frac{\exp\left(-X^2/(4(T-\lambda))\right)}{(T-\lambda)^{3/2}} d\lambda \qquad (1)$$

where:

$$G(T) = e^T \left(\exp\left(\frac{V_j(T)}{V_{th}} \right) - 1 \right), \qquad (2)$$

where V_{th} is the thermal voltage, which at room temperature is about 0.025 V. The factor $\exp(T)$ in (2) is included for convenience.

To find the derivative and thence the current, this can be transformed using a substitution from Carslaw and Jaeger[2] (p. 63):

$$p(X,T) = p_n \frac{2e^{-T}}{\pi^{1/2}} \int_{X/(2\sqrt{T})}^{\infty} G\left(T - \frac{X^2}{4\lambda^2} \right) e^{-\lambda^2} d\lambda \qquad (3)$$

Differentiating yields two terms. The first arises from the lower limit in the integral and is:

$$\left(\frac{dp}{dX} \right)_{X=0}^{(1)} = -p_n \frac{e^{-T} G(0)}{(\pi T)^{1/2}} \qquad (4)$$

(Care must be taken to set $X = 0$ as the last operation.) The second contribution is due to a differentiation of the integrand:

$$\left(\frac{dp}{dX} \right)_{X=0}^{(2)} = -p_n \frac{e^{-T}}{\pi^{1/2}} \int_{X/(2\sqrt{T})}^{\infty} G'\left(T - \frac{X^2}{4\lambda^2} \right) \frac{Xe^{-\lambda^2}}{\lambda^2} d\lambda \qquad (5)$$

Here the prime indicates a derivative with respect to time or in this case λ. Now let $\mu = X^2/(4\lambda^2)$ and this becomes:

$$\left(\frac{dp}{dX}\right)^{(2)}_{X=0} = -p_n \frac{e^{-T}}{\pi^{1/2}} \int_0^T \frac{G'(T-\mu)}{\mu^{1/2}} d\mu \quad (6)$$

Another substitution yields a result roughly in the form presented by Lax and Neustadter:

$$\left(\frac{dp}{dX}\right)^{(2)}_{X=0} = -p_n \frac{e^{-T}}{\pi^{1/2}} \int_0^T \frac{G'(\mu)}{(T-\mu)^{1/2}} d\mu \quad (7)$$

The second term provided by Lax and Neustadter[1] is given by:

$$p(X,T) = \frac{p_n G(0) e^{-T}}{2(\pi T)^{1/2}} \quad (8)$$
$$\times \int_0^\infty e^{-\mu} \left(\exp\left(-\frac{(X-\mu)^2}{4T} \right) - \exp\left(-\frac{(X+\mu)^2}{4T} \right) \right) d\mu$$

Again we need the derivative of the hole density with respect to X at $X = 0$:

$$\left(\frac{dp}{dX}\right)^{(3)}_{X=0} = \frac{p_n G(0) e^{-T}}{2(\pi T)^{1/2}} \int_0^\infty \frac{\mu e^{-\mu} \exp\left(-\mu^2/(4T)\right)}{T} d\mu \quad (9)$$

After rearranging this becomes:

$$\left(\frac{dp}{dX}\right)^{(3)}_{X=0} = \frac{p_n G(0)}{2(\pi T)^{1/2}} \int_0^\infty \frac{\mu}{T} \exp\left(-\left(\frac{\mu}{2T^{1/2}} + T^{1/2} \right)^2 \right) d\mu \quad (10)$$

Using the substitution:

$$z = \frac{\mu}{2T^{1/2}} + T^{1/2}, \quad (11)$$

we find that:

$$\left(\frac{dp}{dx}\right)^{(3)}_{X=0} = p_n G(0) \left(\frac{e^{-T}}{(\pi T)^{1/2}} - \mathrm{erfc}(T^{1/2}) \right) \quad (12)$$

Adding the various components together, we note that two of the terms cancel and our result is:

$$\left(\frac{dp}{dX}\right)_{X=0} = -p_n \frac{e^{-T}}{\pi^{1/2}} \int_0^T \frac{G'(\mu)}{(T-\mu)^{1/2}} d\mu - G(0)\mathrm{erfc}(\sqrt{T}). \quad (13)$$

The corrected equation for the hole current passing from the junction into the n-type material is given by:

$$i(T) = \frac{qDAp_n}{L} \left(\begin{array}{l} \dfrac{e^{-T}}{\pi^{1/2}} \displaystyle\int_0^T \frac{G'(\mu)}{(T-\mu)^{1/2}} d\mu \\ + G(0)\mathrm{erfc}(\sqrt{T}) \end{array} \right), \quad (14)$$

where q is the electronic charge, D is the diffusion coefficient, A is the junction area and L is the diffusion length. This resembles the Lax and Neustadter equation but the square root in the denominator of their integral is missing. Their equation is also dimensionally inconsistent. A current in the reverse direction corresponds to a negative current.

A particular case is when the junction voltage is constant in time and this case is useful for analysis to verify in part the theoretical approach. It corresponds to a situation where the hole density is constant and a constant hole current flows from the junction into the n-type material to maintain it. Thus, we evaluate (13) for constant $V_j(T)$ in (2). Now, G is given by:

$$G(T) = e^T G_0, \quad (15)$$

where G_0 is constant. The derivative of G is:

$$\frac{dG}{dT} = e^T G_0 \quad (16)$$

Inserting this into (13) gives:

$$\left(\frac{dp}{dX}\right)_{X=0} = -\frac{e^{-T}}{\pi^{1/2}} \int_0^T \frac{G_0 e^{\mu}}{(T-\mu)^{1/2}} d\mu - G_0 \mathrm{erfc}(\sqrt{T}) \quad (17)$$

Substituting $z^2 = T - \mu$ gives:

$$\left(\frac{dp}{dX}\right)_{X=0} = -\frac{2G_0}{\pi^{1/2}} \int_0^{\sqrt{T}} e^{-z^2} dz - G_0 \mathrm{erfc}(\sqrt{T}) \quad (18)$$
$$= -G_0 \left(\mathrm{erf}(\sqrt{T}) + \mathrm{erfc}(\sqrt{T}) \right) = -G_0$$

We see that the derivative is indeed constant and negative.

Equations (13) and (14) are not satisfactory from a numerical integration perspective. The derivative and the pole are problematic and it is advantageous to transform to a more convenient form. In fact some efforts were made to employ these equations but the results were unacceptable. Therefore a transformation adopted by Lax and Neustadter was employed. This relies on a solution of Abel's equation, which is described by Whittaker and Watson[3]. Using their notation, the equation is of the form:

$$f(x) = \int_a^x \frac{u(\xi) d\xi}{(x-\xi)^{\mu}}, \quad 0 < \mu < 1 \quad (18)$$

Its solution is:

$$u(z) = \frac{\sin(\pi\mu)}{\pi} \frac{d}{dz} \int_a^z \frac{f(x)dx}{(z-x)^{1-\mu}} \quad (19)$$

Here $f'(x)$ is continuous and $f(a) = 0$. In our case $\mu = ½$ so the sine term is just one. Therefore, if $u = G'$, we find that:

$$G'(T) = \frac{1}{\pi} \frac{d}{dT} \int_0^T \frac{f(x)dx}{(T-x)^{1/2}} \quad (20)$$

This can be integrated to yield:

$$G(T) = \frac{1}{\pi} \int_0^T \frac{f(x)dx}{(T-x)^{1/2}} + K \quad (21)$$

where K is a constant of integration.

The derivative of $G(0)$ and its complementary error function can be brought into the integral. In Ref. 1 this is handled in Appendices II and III. These explain how the integrals necessary to perform the operation in (19) are performed. The integrals are straightforward and it is shown that one of them can be evaluated by a simple integration over a quadrant of a circle: this will not be repeated here. The result is:

$$I_S(G(T) - G(0)e^T \operatorname{erfc}(\sqrt{T})) = \frac{1}{\sqrt{\pi}} \int_0^T \frac{e^{\lambda} i(\lambda)}{(T-\lambda)^{1/2}} d\lambda \quad (22)$$

where I_S is the diode saturation current given by the constant factor in (14). By considering the case of steady forward current[1], it can be seen that $K = 0$.

The current in (22) must also satisfy Ohm's law:

$$i(T) = \frac{V_R - V_J(T)}{R_2}, \quad (23)$$

where V_R is the applied reverse voltage and R_2 is the circuit resistance as in Figure 1. Inserting this into (22) gives:

$$I_S(\exp(V_J(T)/V_{th}) - 1)) = I_f \operatorname{erfc}(\sqrt{T}) + I_R \operatorname{erf}(\sqrt{T}) - \frac{1}{R_2\sqrt{\pi}} \int_0^T \frac{e^{\lambda - T} V_J(\lambda)}{(T-\lambda)^{1/2}} d\lambda, \quad (24)$$

where I_f is the forward current and I_R is V_R/R_2; this is similar to the Lax and Neustadter result.

The integral can be expressed in a more useful form by a simple substitution and we have:

$$I_S(\exp(V_J(T)/V_{th}) - 1)) = I_f \operatorname{erfc}(\sqrt{T}) + I_R \operatorname{erf}(\sqrt{T}) - \frac{2}{R_2\sqrt{\pi}} \int_0^{\sqrt{T}} \exp(-z^2) V_J(T - z^2) d\lambda, \quad (25)$$

Therefore, we must determine the junction voltage $Vj(T)$ and this is achieved using a bisection search based on the currents in (25). The search is confined to a specific range of V_J and the lower limit is the applied reverse voltage, while the upper limit is the sum of the junction voltage just after the switch is thrown and the magnitude of the applied reverse voltage. The junction voltage as a function of time is stored as an array and this permits a simple calculation of the integral in (25).

The parameters of the diode used by Lax and Neustadter[1] are provided in Table I. The characteristic diffusion length, $L = (D\tau)^{1/2}$. The reverse currents for the diode with R_2 = 2000 Ω, 1000 Ω and 500Ω are shown as a function of time in Figure 2. We also include the case of zero resistance; here the junction voltage falls to the applied voltage immediately the switch is thrown. The current, normalized to the initial forward current (this latter is given by the product of the first factor in (14) with $G(0)$) has been established independently (e.g. Kingston[3]) as:

$$I(T) = \operatorname{erfc}(\sqrt{T}) - \frac{e^{-T}}{\sqrt{\pi T}} \quad (21)$$

This is not a function of V_R, because the current is controlled almost exclusively by diffusion and recombination. (The reverse current, which is shown in Figure 2, is the negative of this.)

Table I. Diode Parameters

Parameter	Value
Diffusion Coefficient, D	44 cm^2/s
Junction Area, A	0.0025 cm^2
Forward Current, I_f	6.0 mA
Equilibrium Density, p_n	2.5x10^{12} cm^{-3}
Recombination Time, τ	100 μs
Reverse Voltage, V_R	-6 V

The graphs, which are calculated directly from (25), are quantitatively similar to those shown in Lax and Neustadter's paper.

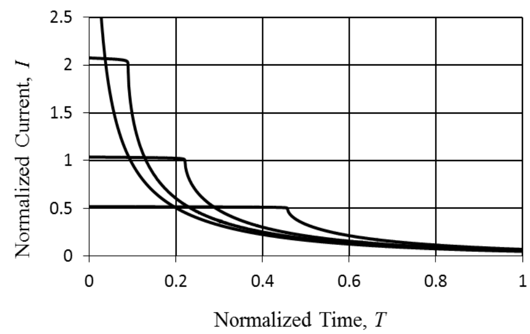

Figure 2. Reverse current. From lowest to highest plots R_2 = 2000 Ω, 1000 Ω, 500 Ω, 0 Ω.

Figure 3 shows the junction voltage, normalized to the initial voltage of 0.228 V, as a function of time for the same circuit resistances except for 0 Ω. As expected, the recovery times are similar to those in Figure 2. At T = 0, this voltage is 1.0 and at T = 1 has a value of just less than -25. After a very long time we expect the value to be -6.0 / 0.228 = -26.3.

In summary, the new direct numerical approach appears to provide reasonable results, which are consistent with accepted analytical results for asymptotic behavior at long times. These results are also consistent with those described by Lax and Neustadter[1].

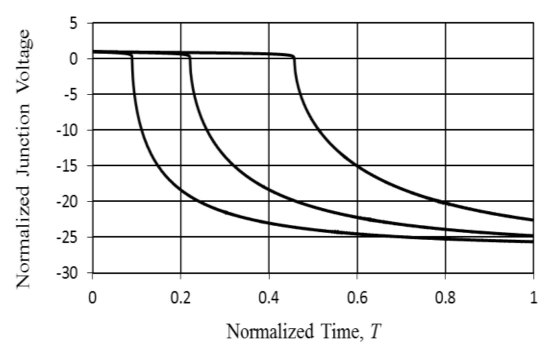

Figure 3. Junction voltage normalized to initial value; R_2 = 500, 1000 and 2000 Ω.

The theory is easily extended to handle a sinusoidal or other excitation. For the sinusoid, (25) is modified and becomes:

$$I_S(\exp(V_J(T)/V_{th})-1)) = \frac{2}{R_2\sqrt{\pi}}\int_0^{\sqrt{T}} e^{-z^2}\left(V_0 \sin(2\pi f(T-z^2)) - V_J(T-z^2)\right)dz \quad (25)$$

where V_0 is the amplitude of the applied voltage and f is a normalized frequency. The result for a frequency that corresponds to a period that is half of the recombination period with V_0 = 1 V and R_2 = 50Ω is shown in Figure 4. Transient effects are just about visible.

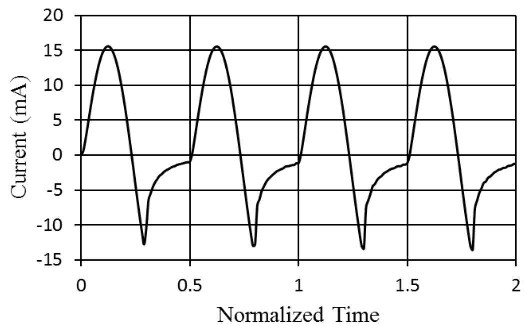

Figure 4. Response to sinusoidal signal.

1. B. Lax and S.F. Neustadter, "Transient response of a p-n junction", *J. Appl. Phys*. 25, 1148-1154 (1954).

2. H.S. Carslaw and J.C. Jaeger, *Conduction of Heat in Solids*, 2nd Ed., Oxford Clarendon Press (1959).

3. E.T. Whittaker and G.N. Watson, *A Course of Modern Analysis*, Cambridge, 1965.

4. R.H. Kingston, "Switching time in junction diodes and junction transistors", *Proc. IRE* 42, 829-834 (1954).