# Wakes from Go-Fast and Small Planing Boats

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*Abstract*—**In both optical and radar images, the wakes from high speed planing boats seem to be significantly narrower than the Kelvin wake. It has been conjectured that the classical Kelvin wake explanation of the crest pattern is insufficient because the effect is due to the absence of waves of long wavelength generated by ships of small length traveling at high speed. This paper describes simulations of the Kelvin wakes and demonstrates that both the accepted, full Kelvin theory and the conjecture are reasonable.**

*Index Terms*—**Go-Fast Boat, Ship Wake, Kelvin Wake.**

## I. INTRODUCTION

SHIP wakes on the ocean are commonly observed in both optical and high resolution radar imagery. The interpretation of these images requires a proper understanding in applications such as ocean surveillance. Perhaps the classical Kelvin wake, with a half angle of 19.47º, is the most well-known and most prominent in optical images; this corresponds to the wake created by a ship moving on calm deep water at constant velocity [1].

Wakes have been observed in Google images from optical sensors by Rabaud and Moisy [2] and their angles are sometimes much less than that of the classical Kelvin wake. These wakes have been called "narrow-V wakes". It turns out that the wake angle tends to fall off as the Froude number increases above a value of about 0.6. Eventually the wake angle becomes inversely proportional to the Froude number. The authors suggest that the wakes from small high-speed craft do not contain significant waves with a large wavelength and that the maximum wavelength is limited by the length of the boat.

A very simple model, which limits the spectrum of the ship's excitation, seems to confirm this (as might be expected). However, this model is not realistic.

In a further study Darmon et al. [3] simulate the wakes created by a moving pressure distribution as might be produced by a hovercraft. However, the pressure distribution over the surface is Gaussian, which again implies that the excitation spectrum is artificially limited. It is found that narrow wakes are produced at high Froude numbers and this is more or less consistent with [2].

Noblesse et al. [4] discuss the implications of Kelvin theory for both monohulls and catamarans. They argue that at high speed divergent waves created by different parts of the hull (specifically the contributions from near the bow and near the stern) can interfere to produce bright wake lines at narrow angles. The same mechanism can cause the transverse structure to almost cancel. This cancellation does not occur with pressure distributions of the type discussed in [3]. Therefore the explanation provided in [4] is very plausible.

The present paper employs simulations to confirm that narrow-V wakes can indeed occur for normal planing craft and go-fast boats and to identify the conditions under which a suppression of the long waves in the Kelvin wake occurs. Realistic hull models are employed and the excitation spectrum is not artificially cut-off.

Planing power boats designed for offshore racing were introduced in the early 1900s. During prohibition they were used for smuggling liquor and later cigarettes. Stability and maneuverability were greatly improved during the 1960s and these boats evolved into affordable pleasure craft that can be found on rivers, lakes as well as in coastal areas. Go-fast boats are still employed for smuggling especially in the Caribbean. Theory for the performance of planing boats has been developed by Savitsky [5] from empirical data.

A typical small pleasure power boat can travel at 70 km/hr or more and its length is upwards of 5 m. Go-fast boats tend to be somewhat longer and slimmer and speeds can exceed 100 km/hr even in choppy water.

Kelvin wake theory has a long history. The opening angle of the wake crest pattern is independent of speed and the pattern is fixed apart from a scale factor that depends on the speed. However, it cannot be assumed that all Kelvin wakes have the same general appearance because the wave amplitudes depend on the hull size and shape as well as its speed.

The effect of the hull shape was reported in some detail by Havelock [6] in 1934. At that time planing boats were not common so that their characteristics were not explicitly addressed. Havelock showed that for all hulls the wake amplitudes are controlled by a few principal factors, namely the speed, length and draft of the ship. Each part of the hull generates a disturbance that is constant in the reference frame of the hull. These disturbances combine to determine the wave amplitudes in each part of the wake, which in calm water also maintains a constant shape in the reference frame of the ship. Interference between waves due to disturbances from different locations on the hull greatly affects the overall wake appearance. The effect of the ship length and its speed is summarized by the usual Froude number.

Another factor is the ship draft. Waves generated from parts

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of the hull at large drafts are attenuated at the surface. The attenuation falls as the wavelength of the wave increases. The transverse wave structure of the Kelvin wake comprises the longest waves in the wake so that these tend to be prominent for large, deep-draft, ocean-going cargo vessels. Relative to the transverse waves, the small wavelength waves in the divergent wave structure tend to be suppressed.

Thus Havelock showed that large deep draft commercial ships traveling at typical service speeds tend to generate wakes with a prominent transverse structure. Small, shallow draft craft moving at high speed tend to produce wakes with prominent divergent wake structure and only small amplitude transverse waves.

Ship hulls can be placed into three categories, namely displacement vessels, semi-displacement vessels and planing craft [7]. Displacement vessels float by displacing water to create a buoyancy force that cancels gravity. Semi-displacement vessels augment the buoyancy force by hydrodynamic pressure due to partial planing. Planing vessels are supported vertically mainly by hydrodynamic pressure due to water flowing under the hull [5]. As a planing vessel increases its speed, the bow rises and the bottom adopts a small angle of attack called the trim angle. Water is forced downwards and this creates both a vertical force component to support the ship and a horizontal drag component. As the speed increases further, the trim angle decreases and the area of the hull in contact with the water moves sternwards. The hull typically ends abruptly in a transom stern.

A small hull area in contact with the water is advantageous because it reduces both the frictional drag, which is due to viscosity, and wave-making resistance. This allows the craft to attain high maximum speeds using engines of moderate power.

## II. KELVIN WAKE SIMULATION

Simulation is based on the theory described by Wehausen and Laitone [8] and to some extent by Tuck et al. [9]. Various approaches are possible; the one pioneered by Havelock is used here. However, there appear to be significant variations in the quoted formulae with regard to constants. Therefore the theory was re-derived from first principles and we note that the end result is consistent with that quoted by Noblesse [10] in his equation (10). In the evaluation of the integrals, a stationary phase approach is adopted and great care is exercised in this study that only wake waves that are physically reasonable are included.

Calculations based on stationary phase become quite accurate in the far wake. It should be noted that the domain in which stationary phase methods are accurate can be unexpectedly large so that, though the approach is strictly applicable to the far wake, it is not wildly incorrect even close to the ship.

The caustic, which occurs at the wake edge where the divergent and transverse waves coalesce, is handled using Airy functions in a manner described by Lighthill [1].`

The calculation of the wave amplitudes is accomplished by representing the hull as a distribution of isotropic sources and sinks of fluid [7]. The sources and sinks (a sink is just a negative source) are chosen so that the flow field from them (outside of the ship) is as close as possible to the actual flow around the hull. Therefore in the theory, the hull can be replaced by the distribution. The remaining problem is to find the wave amplitudes from a single point source moving horizontally at constant velocity and then to integrate the result over the distribution. This process is simplified if a linear approximation to the surface perturbation is employed.

To illustrate the effects on the Kelvin wake, it is not necessary to invoke an accurate, complicated hull-model and the "thin ship" approximation is used. In this approach, the flows around the hull are modeled by placing the sources and sinks on the vertical longitudinal centreplane of the hull.

Because the flow around a ship does not usually involve the creation or destruction of fluid, the sum of the sources and sinks should be zero. The allocation of their distribution is made complicated by the fact that the water is not an ideal fluid and this implies that a boundary layer is created around the submerged hull. This boundary layer tends to separate from the hull at or near the stern and creates a turbulent wake. Separation tends to result in a reduction of pressure near the stern, which results in form drag. To maintain speed, the various drag contributions must be compensated by thrust from a propeller. In principle, the turbulent wake should include the flows due to the propeller.

The representation of a ship hull and its turbulent wake has been addressed by Havelock [11], who recommends reducing the sink inputs at the stern relative to the source outputs of fluid near the bow by a factor, such as 0.6. This implies that some sources and sinks could be located at infinity. Thus it cannot be assumed that the additional flow associated with the turbulent wake (in the direction of the ship) is exactly compensated by the flow generated by the propeller (in the sternward direction). The source strength integrated over the hull is not necessarily zero.

Havelock and Tuck et al. perform integrals over the source distribution to find the wave pattern amplitudes in the far field but here the amplitudes from a unit source are calculated and the wave patterns are summed. This is a superior approach because it gives a better approximation to the near field and, as will be seen, it can show vestiges of the Kelvin arms that are omitted in the former technique.

## III. HULL MODELS

The basic model is a wedge shape appropriate to the submerged part of a planing hull at high speed [5]. In the transverse plane, the hull is V-shaped with a deadrise angle, $\delta$, of 10º to 20º. The offsets during planing are shown in Fig. 1. This is simplified so that planing takes place entirely on the V-shape and the submerged volume is in the form of a wedge. The length of the wedge, $L_{\text{eff}}$, is typically about two thirds of the length of the craft. The maximum width of the wedge is approximately equal to the beam, $B$, and the draft, $D$, is typically of the order of a few centimeters.

The chine is an abrupt edge where the hull changes its slope; it helps to prevents spray from entering the boat. Above the chine, the hull rises almost vertically. Fig. 1 is exaggerated.

The bottom of the hull is also characterized by its trim angle, $\tau$, which is typically between 2° and 15° [7] [14]. These can be related to the nominal dimensions of the submerged volume:

$$\begin{aligned} \tan\delta &= 2D/B \\ \tan\tau &= D/L_{eff} \end{aligned} . \tag{1}$$

Proposed dimensions should result in reasonable values for these angles.

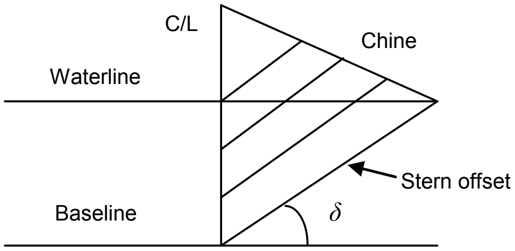

Figure 1. Notional planing craft offsets.

The slope of the hull in the horizontal direction at constant depth, which is used in the source distribution, is independent of depth. At the waterline area, it is evident that this slope is equal to $B/(2L_{eff})$. The only difference between the horizontal slices is their length and their starting position from the bow. The starting position of a slice is a linear function of depth and depends on the trim; all slices terminate at the transom stern.

The model for the planing hull is based roughly on the Bayliner 180, which is a small pleasure craft. The approximate measured dimensions and other information derived in part from the operators manual are shown in Table 1. From (1), the maximum draft during planing should be about 0.36 m. The deadrise angle is consistent with (1). Also from (1), the maximum trim angle should be about 5°.

TABLE 1.
PLANING BOAT PARAMETERS

| Parameter | Value |
|---|---|
| Overall Length (m) | 5.5 |
| Beam (m) | 2.3 |
| Deadrise (°) | 17 |
| Dry Mass (kg) | 850 |
| Loaded Mass (kg) | 1400 |

TABLE 2.
DERIVED PARAMETERS

| Parameter | Value |
|---|---|
| Speed (m/s) | 10.0 |
| LCG (m) | 2.0 |
| Trim (°) | 4.2 |
| Mean Planing Length, $L_{eff}$ (m) | 2.9 |
| Planing Area (m$^2$) | 7.0 |
| Effective Propeller Power (kW) | 18.9 |

To estimate the trim angle, the Savitsky equations in [5] were employed using an approach described in [7]. This also included other parameters including the wetted area during planing and the effective power supplied by the propeller. Fresh water was assumed. These data are provided in Table 2 for a loaded boat traveling at a speed of 10 m/s and assuming that the distance between the transom and the Centre of Gravity, LCG, is 2.0 m.

As the speed increases, the trim angle and the planing length decrease, while the effective propeller power increases. The Savitsky equations are valid only for trim angles greater than 2° and this limits the speed in the calculation. Results are shown in Table 3.

TABLE 3.
TRIM ANGLES

| Speed (m/s) | Trim (°) | $L_{eff}$ (m) | Power (kW) |
|---|---|---|---|
| 10.0 | 4.2 | 2.90 | 18.9 |
| 12.5 | 3.2 | 2.81 | 26.0 |
| 15.0 | 2.5 | 2.77 | 36.4 |
| 17.5 | 2.1 | 2.74 | 50.8 |

For comparison, the hull offsets of single-propeller cargo ships are shown in Fig. 2. This model was developed at the David Taylor Model Basin and is known as the "DTMB Series 60" [12]. The DTMB hull offsets are for a hull with a block coefficient, $C_B$, of 0.6.

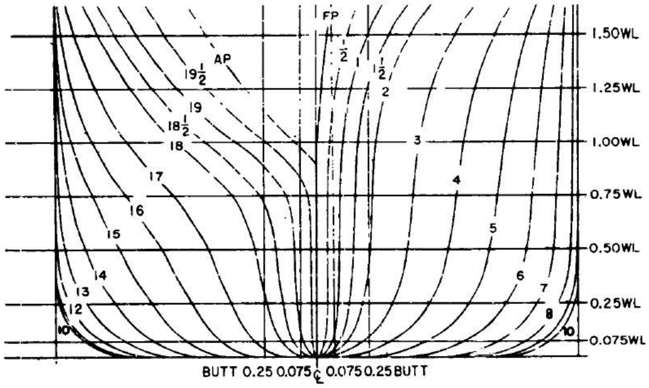

Figure 2. DTMB Series 60 offsets ($C_B$ = 0.60; from Todd 1963). WL = Waterline; FP = Forward Perpendicular; AP = Aft Perpendicular. Bow section is on the right and stern on the left.

## IV. SIMULATION DETAILS

The crest pattern in the Kelvin wake can be described in terms of dimensionless quantities. The dimensionless angular wave number, $\kappa$, can be expressed in terms of the actual angular wave number, $k$:

$$\kappa = kU^2/g , \tag{2}$$

where $g$ is the acceleration due to gravity.

For the planing hull, a characteristic length can be defined as the distance between the effective position of the sources representing the submerged hull and the propeller. The distance between the transom stern and the propeller is about 0.5 m. Therefore the distance between a source and the propeller lies between 0.5 m and 3.5 m. This implies a range of characteristic values of $\kappa$ and, at a speed of 15 m/s, this is 33 to 288. Thus the wavelengths of transverse and cusp waves, which are associated with values of $\kappa$ near unity (e.g. [2] [13]), are much greater than that of the typical waves generated by the hull; they will be suppressed.

Similar conclusions are obtained using the Froude number, $F_L$. This is defined as usual by:

$$F_L = U / \sqrt{gL} , \tag{3}$$

The effect of the draft can be summarized by another Froude number based on the ship draft rather than its length, i.e.

$$F_D = U / \sqrt{gD} \tag{4}$$

This is very large in the present context and indicates that attenuation of waves created at depth can be ignored.

Because the Froude numbers are high, it is tempting to replace the source distribution by a single source just under the surface located at a short distance from the stern. A divisor of 4 should be applied to the planing length because the submerged volume is wedge-shaped in both depth and beam. In the case of a planing boat with transom stern, there is little back-flow at the stern and sinks are not required here. However, the fluid flow is affected by the propeller, which must compensate wave-making resistance, spray drag and viscous drag by its thrust. The propeller is usually located at a depth just slightly greater than the draft of the boat. To model the overall flow around the boat it seems reasonable to locate a single sink at the propeller position.

The waterline area at planing is approximately triangular but in practice there is a short parallel section at the stern, which will be ignored.

Simulations appropriate to cargo ships show the expected structure and these wakes will not be pursued here.

## V. RESULTS

At first it is assumed that the sum of the sources and sinks of the planing hull is zero. The result for the Bayliner at a speed of 15 m/s is shown in Fig. 3. The pixel size is 0.1 m and the length of the image is 150 m. Five horizontal slices were used for the depth. The maximum wake amplitude is just over 0.3 m. If the sinks are omitted entirely, the result is shown in Fig. 4. There are differences but these are not marked and the two wakes are qualitatively similar but the first wake is a little narrower. The maximum wake amplitude is again about 0.3 m.

When the model source and sink distribution is replaced by a single source and sink pair, the simulated wake is quite similar to Fig. 3 but the maximum wake amplitude is now somewhat less than 0.3 m.

Fig. 5 shows the wake from a go-fast boat traveling at 30 m/s. It is assumed that the sources and sinks cancel. The planing length is 10 m and the beam is 2.5 m. It is assumed that the draft is 0.25 m. The maximum amplitude of the wake is somewhat greater than 0.3 m.

When the sources and sinks cancel, faint Kelvin arms are visible.

## VI. DISCUSSION

Kohansal et al. [14] have simulated the Kelvin wake from a planing vessel. This was implemented by discretizing both the hull and the free surface and solving a Green's function problem numerically. The simulation is limited to a small area but the wake seems to exhibit a much smaller angle than that of the classical Kelvin arms as has been demonstrated here.

Narrow wakes are produced even when the sources at the bow are not cancelled by sinks at the stern. However, when partial or complete cancellation occurs, the wake tends to be confined to a narrower angle. The extent of cancellation does not appear to be a critical factor.

When there is cancellation, the present simulations show faint vestiges of the Kelvin arms because the waves from sources at the bow are not cancelled by waves from sinks at the stern in the most forward part of the wake.

## VII. CONCLUSION

Narrow-V wakes have been simulated using standard Kelvin wake theory. The principal factor seems to be the excitation spectrum as suggested in [2] but cancellation of sources by sinks enhances the effect as proposed in [4]. The extent of cancellation does not seem to be critical.

As expected, the wakes tend to narrow with increasing Froude number, $F_L$.

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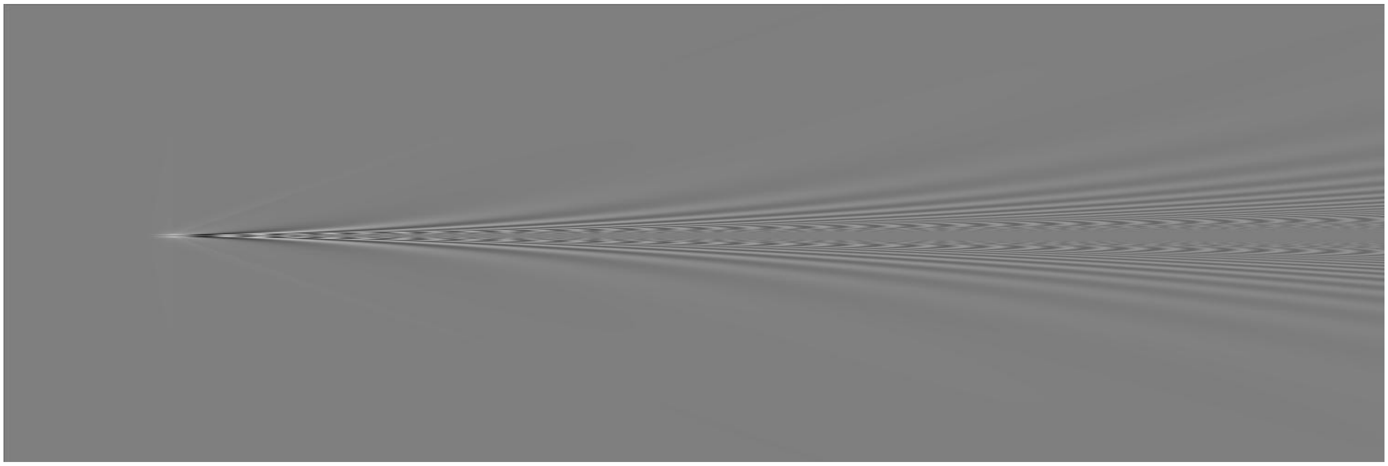

Figure 3. Wake from a planing boat at 15 m/s using data from Tables 1-3.

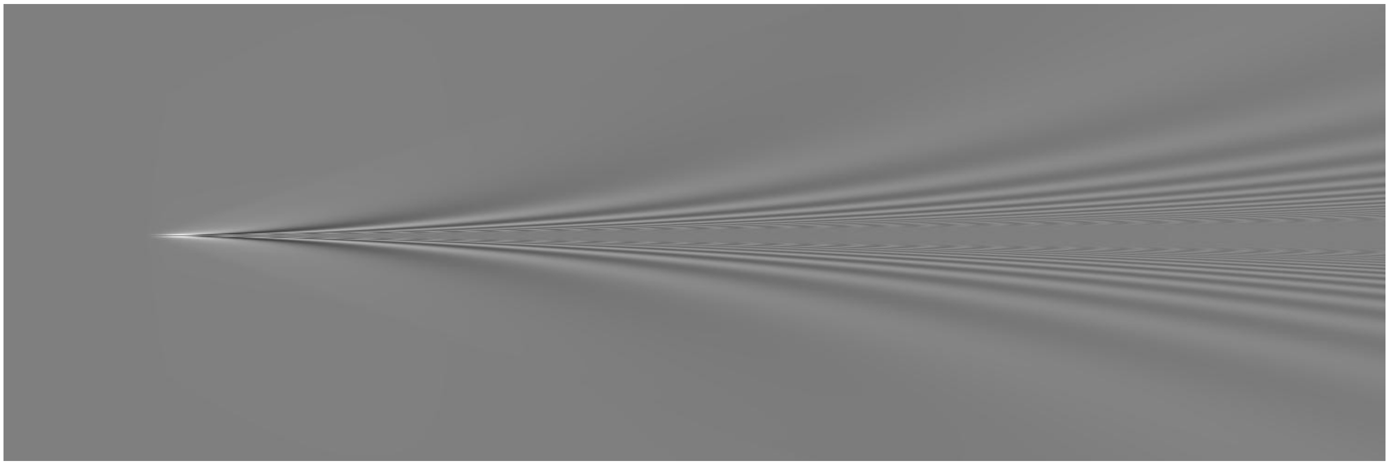

Figure 4. Similar wake from a planing boat at 15 m/s but with sinks omitted.

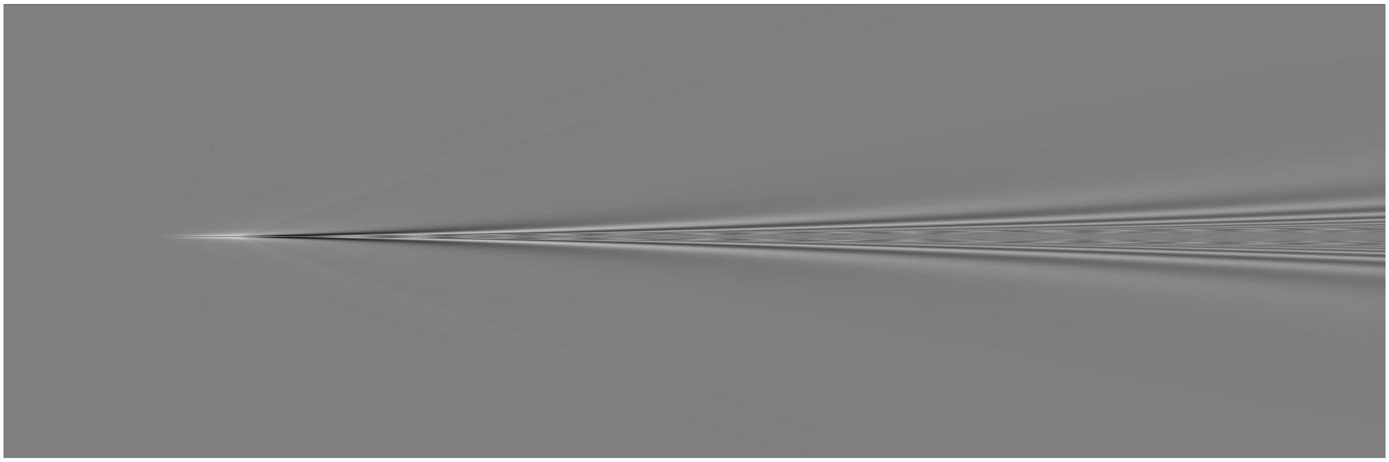

Figure 5. Wake from go-fast boat at 30 m/s.