SAR VELOCITY BUNCHING RELATIONSHIPS

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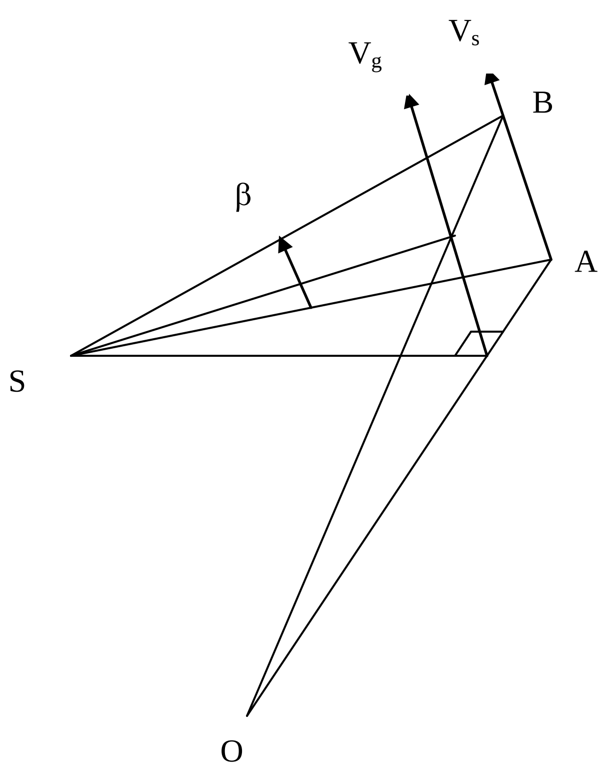
When calculating the relation between the azimuthal shift in the image plane and the line of sight velocity of the target in the direction of the radar, account must be taken of the curvature of the satellite orbit and of the earth. In this rather simple derivation, the rotation of the earth is neglected because it is reasonably small compared with satellite speeds and we are primarily concerned with polar orbits.

Because the azimuthal resolution of a side-looking Synthetic Aperture Radar (SAR) relies on the phase history of the returns from a scatterer, S, the position of the scatterer in the image plane can be related to the Doppler shift. A stationary scatterer located behind the satellite has a Doppler frequency that is reduced while a scatterer ahead of the satellite will produce an increase in the Doppler frequency at the satellite receiver. The motion of a scatterer away from the satellite will be interpreted as if the scatterer were shifted in azimuth or cross-range to a position behind its true position (satellite at B). This is shown in the following diagram (AB is perpendicular to AS):



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The following definitions are used:

V_s = Satellite velocity
V_g = Satellite velocity over the ground
R_s = Slant range (SA)
v = Doppler velocity (along AS if S is moving or BS if S is stationary)
t = Time corresponding to satellite motion over AB
r = Azimuthal shift
O = Earth centre
β = Angle between SA and SB

From the geometry, for small angles, β, and equating the Doppler velocities for S in motion or not, we have:

$$\beta = \frac{V_s t}{R_s}; \quad v = V_s \beta; \quad r = V_g t$$

(Note that in SAR we might consider that factors of 2 occur in the middle equation but on both sides of it, so they cancel out.) Solving these equations for r yields:

$$r = v \frac{R_s V_g}{V_s^2}$$

Therefore, if we introduce an effective satellite velocity equal to V_s^2/V_g, we have the final result $r = vR_s/V_{eff}$. For the flat earth approximation, appropriate to airborne SAR, $V_s = V_g$ and this simplifies to a well-known result involving just the aircraft velocity.

The magnitudes of V_s and V_g differ significantly in practice so that the effect is important, for example when a ship's position in a SAR image must be related to its wake or to Automatic Indentification of Ships (AIS) self-reporting.

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